

# Obtaining $|V_{ub}|$ exclusively: a theoretical perspective

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## 1 Introduction

Recent inclusive determinations of  $|V_{ub}|$  have uncertainties of approximately 10% [1], as opposed to  $\lesssim 2\%$  on  $|V_{cb}|$  via  $B \rightarrow X_c l \nu$  [2]. Exclusive channels provide a competitive alternative route to  $|V_{ub}|$ , but although experimentally more promising this requires information about hadronic matrix elements via form factors. Form factors are calculable via non-perturbative techniques such as Lattice QCD (see e.g. refs. [3]) or QCD sum rules on the light-cone (LCSR). Predictions are usually confined to a particular region of  $q^2$ , the momentum transfer squared, i.e. LCSR and Lattice are restricted to large and small recoil energies of the daughter hadron respectively. In LCSR one considers a correlator  $\Pi_\mu$  of the time-ordered product of two quark currents, sandwiched between the final state hadron, which is on shell, and the vacuum [4], i.e. for a  $B$  decaying to a  $\pi$  of momenta  $p_B$  and  $p$ ,

$$\Pi_\mu = i m_b \int d^D x e^{-i p_B \cdot x} \langle \pi(p) | T \{ \bar{u}(0) \gamma_\mu b(0) \bar{b}(x) i \gamma_5 d(x) \} | 0 \rangle. \quad (1)$$

This can be expressed on one hand by a light-cone expansion via perturbative hard scattering kernels convoluted with non-perturbative light-cone distribution amplitudes (LCDAs), ordered in increasing twist, or by inserting a sum over excited states, i.e. the  $b$  hadron and a continuum of heavier states. Assuming quark hadron duality above a certain continuum threshold  $s_0$ , one can subtract this continuum contribution from both sides. Borel transforming this relation then ensures that this assumption, and the truncation of the series, have a minimal effect on the resulting sum rule. At present,  $|V_{ub,excl}|$  is obtained most precisely from  $B \rightarrow \pi l \nu$ , where in the limit of massless leptons the decay rate for  $B \rightarrow \pi$  depends on a single form factor  $f_+(q^2)$ . However by considering other channels, e.g. baryonic decays such as  $\Lambda_b \rightarrow p l \nu$ , one can obtain interesting complementary information.<sup>1</sup> Here I will discuss recent progress in the calculation of the form factors for  $B \rightarrow \pi l \nu$  [9] and  $\Lambda_b \rightarrow p l \nu$  [8] using LCSR.

<sup>1</sup>In the limit of massless leptons the decay rate for  $\Lambda_b \rightarrow p l \nu$  depends on four form factors,  $f_{1,2}(q^2)$  and  $g_{1,2}(q^2)$ .

## 2 Recent LCSR updates on $f_+(q^2)$ for $B \rightarrow \pi l \nu$

There has been much progress in the LCSR calculations of  $f_+(q^2)$  in the last 15 years. The next-to-leading order (NLO) corrections to  $f_+(q^2)$  at leading twist (twist-2) were first calculated in LCSR in ref. [5] and LO corrections up to twist-4 were calculated in ref. [6]. Since the LO twist-3 contribution was found to be large, it was confirmed that the NLO corrections are under control, using both the pole and  $\overline{\text{MS}}$  mass for  $m_b$  [7]. In ref. [8], different values for the moments of the twist-2 LCDA were employed, extracted from latest experimental data for  $F_\pi$  using LCSR. The normalised decay rate integrated over a given range in  $q^2$ ,

$$\Delta\zeta(0, q_{\text{max}}^2) = \frac{1}{|V_{ub}|^2} \int_0^{q_{\text{max}}^2} dq^2 \frac{d\Gamma}{dq^2}(\Lambda_b \rightarrow p l \nu), \quad (2)$$

was then predicted to be  $\Delta\zeta(0, 12\text{GeV}^2) = 4.59_{-0.85}^{+1.00} \text{ps}^{-1}$ , which can be combined with experimental predictions, allowing the extraction of  $|V_{ub}|$ .

Two-loop corrections to the form factor  $f_+(q^2)$  at twist-2 were recently calculated in ref. [9]. In light of the large two-loop sum rules corrections to  $f_B$  calculated in ref. [10], one aim of this work was to test the argument that, in obtaining  $f_+(q^2)$  via LCSR, radiative corrections to  $f_+ f_B$  and  $f_B$  should cancel when both calculated in sum rules. Due to the technical challenges posed by a full calculation, a subset of two-loop radiative corrections for twist-2 contribution to  $f_+(0)$  proportional to  $\beta_0$  was considered, as this gauge invariant subset is thought to be a good approximation to the complete next-to-next-to-leading order (NNLO) result. In combination with the experimental result for  $f_+(0)|V_{ub}|$  one can then obtain  $|V_{ub}|$ . The necessary diagrams are obtained by inserting a fermion bubble in the gluon propagator of the NLO twist-2 diagrams, further details can be found in ref. [9]. The results for  $f_+(0)$ , seen in fig. 1, show that despite the  $\sim 9\%$  positive NNLO corrections to the QCD sum rules result for  $f_B$ , the LCSR prediction for  $f_+(0)$  is stable, increasing by  $\sim 2\%$  to  $f_+(0) = 0.261_{-0.023}^{+0.020}$ , as shown in fig. 1. This enforces the stability of LCSR with respect to higher order corrections, and could be taken to provide confirmation that  $f_B$  from sum rules, not Lattice should be used here. A recent analysis by BaBar [11] finds  $|V_{ub}| = (3.34 \pm 0.10 \pm 0.05 +_{-0.26}^{+0.29})10^{-3}$  using this result, and  $|V_{ub}| = (3.46 \pm 0.06 \pm 0.08 +_{-0.32}^{+0.37})10^{-3}$  using  $\Delta\zeta(0, 12\text{GeV}^2)$  from ref. [8], which are clearly in good agreement.

## 3 Improvements on form factors for $\Lambda_b \rightarrow p$ decays

Recently there has been increasing work on extracting  $|V_{ub}|$  via  $\Lambda_b \rightarrow p l \nu$ . A number of complications arise in LCSR when baryons are considered instead of mesons, the first being the choice of the heavy-light baryon interpolating current  $\eta$  described by

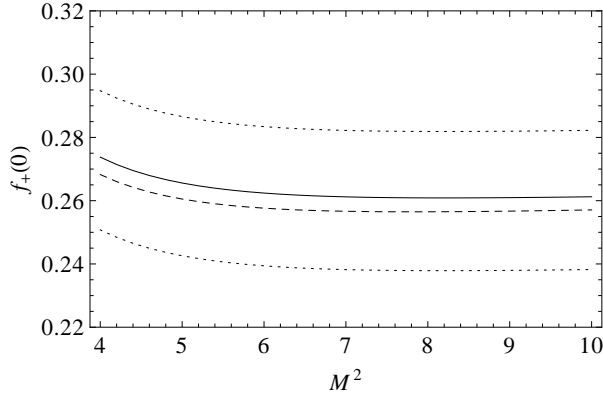


Figure 1:  $f_+(0)$  at  $\mathcal{O}(\alpha_s^2\beta_0)$  for central values of input parameters (solid) with uncertainties (dotted), compared to the  $\mathcal{O}(\alpha_s)$  result calculated using  $s_0 = 34.3\text{GeV}^2$  (dashed), as a function of the Borel parameter  $M^2$ .

$\Gamma_b$  and  $\tilde{\Gamma}_b$ ,

$$\eta = \epsilon^{ijk}(u_i C \Gamma_b d_j) \tilde{\Gamma}_b c_k, \quad (3)$$

debated since the 1980s. Additionally, the contribution of the negative parity  $\Lambda_b^*$  baryon, with  $J^P = 1/2^-$ , which has a similar mass to  $\Lambda_b$  is difficult to isolate, and in the literature was often included in the continuum [12]. Recently however it was found to be possible to separate the  $\Lambda_b^*$  from the  $\Lambda_b$  contribution in the sum rule, and on comparing results for both  $\Gamma_b = \gamma_5(\gamma_5\gamma_\lambda)$  and  $\tilde{\Gamma}_b = 1(\gamma_\lambda)$ , it was found that the resulting form factors show a reduced dependence on the choice of  $\Gamma_b$  and  $\tilde{\Gamma}_b$  [13].

## 4 Summary and Outlook

Recent progress on the LCSR calculation of form factors for the exclusive determination of  $|V_{ub}|$  was presented. This included recent updates on  $f_+(q^2)$ : the 2011 NLO analysis in the  $\overline{\text{MS}}$  scheme resulted in  $|V_{ub}| = (3.46 \pm 0.06 \pm 0.08_{-0.32}^{+0.37})10^{-3}$  and the 2012  $\mathcal{O}(\alpha_s^2\beta_0)$  result found a  $\sim 2\%$  increase in  $f_+(0) = 0.262_{-0.023}^{+0.020}$ , such that  $|V_{ub}| = (3.34 \pm 0.10 \pm 0.05_{-0.26}^{+0.29})10^{-3}$ . New results for the form factors for  $\Lambda_b \rightarrow p l \nu$  were also discussed, where it was showed that by isolating and removing the negative parity baryons' contribution, the form factors show a reduced dependence on the choice of  $\Gamma_b$  and  $\tilde{\Gamma}_b$ . Future work should focus on combining the  $\mathcal{O}(\alpha_s^2\beta_0)$   $f_+(0)$  and Lattice results to determine  $|V_{ub}|$  and calculating remaining twist-2 NNLO corrections to  $f_+(q^2)$  and gluon radiative corrections to the  $\Lambda_b \rightarrow p$  form factors.

## References

- [1] J. P. Lees *et al.* [BABAR Collaboration], Phys. Rev. D **86** (2012) 032004 [arXiv:1112.0702 [hep-ex]], P. Urquijo *et al.* [Belle Collaboration], Phys. Rev. Lett. **104** (2010) 021801 [arXiv:0907.0379 [hep-ex]].
- [2] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **81** (2010) 032003 [arXiv:0908.0415 [hep-ex]], A. Limosani *et al.* [Belle Collaboration], Phys. Rev. Lett. **103** (2009) 241801 [arXiv:0907.1384 [hep-ex]].
- [3] J. A. Bailey, C. Bernard, C. E. DeTar, M. Di Pierro, A. X. El-Khadra, R. T. Evans, E. D. Freeland and E. Gamiz *et al.*, Phys. Rev. D **79** (2009) 054507 [arXiv:0811.3640 [hep-lat]].
- [4] I. Balitsky, V. Braun and A. Kolesnichenko, Nucl. Phys. B **312** (1989) 509.
- [5] A. Khodjamirian, R. Ruckl, S. Weinzierl and O. I. Yakovlev, Phys. Lett. B **410** (1997) 275 [arXiv:hep-ph/9706303];  
E. Bagan, P. Ball and V. M. Braun, Phys. Lett. B **417** (1998) 154 [arXiv:hep-ph/9709243].
- [6] A. Khodjamirian, R. Ruckl, S. Weinzierl, C. W. Winhart and O. I. Yakovlev, Phys. Rev. D **62** (2000) 114002 [arXiv:hep-ph/0001297].
- [7] P. Ball and R. Zwicky, Phys. Rev. D **71** (2005) 014015 [arXiv:hep-ph/0406232],  
G. Duplancic, A. Khodjamirian, T. Mannel, B. Melic and N. Offen, J. Phys. Conf. Ser. **110** (2008) 052026.
- [8] A. Khodjamirian, T. Mannel, N. Offen and Y. M. Wang, Phys. Rev. D **83** (2011) 094031 [arXiv:1103.2655 [hep-ph]].
- [9] A. Bharucha, JHEP **1205** (2012) 092 [arXiv:1203.1359 [hep-ph]].
- [10] M. Jamin and B. O. Lange, Phys. Rev. D **65** (2002) 056005 [arXiv:hep-ph/0108135].
- [11] J. P. Lees *et al.* [BABAR Collaboration], arXiv:1208.1253 [hep-ex].
- [12] Y. -M. Wang, Y. -L. Shen and C. -D. Lu, Phys. Rev. D **80** (2009) 074012 [arXiv:0907.4008 [hep-ph]], K. Azizi, M. Bayar, Y. Sarac and H. Sundu, Phys. Rev. D **80** (2009) 096007. [arXiv:0908.1758 [hep-ph]]
- [13] A. Khodjamirian, C. Klein, T. Mannel and Y. -M. Wang, JHEP **1109** (2011) 106 [arXiv:1108.2971 [hep-ph]].